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Relativistic phase-locked cavities as particle models

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Abstract. The properties of a phase-locked cavity are investigated using special relativity. Some useful formulae are derived and Compton back-scatter is explained. The paper concludes with tentative suggestions for the application of the principles to other phenomena.

1. Introduction

In a recent paper Jennison and Drinkwater (1977) gave a new account of the origin of inertia in which they derived Newton's laws from the properties of radiation trapped in a phase-locked cavity. They investigated the effect of external electromagnetic radiation falling upon the outer wall of a lossless cavity. The force on the wall was found to be initially velocity dependent but the Doppler-shifted radiation reflected from it to the wall at the far end of the cavity caused that end also to be pushed and to reflect back radiation less energetic than that originally in the cavity. If the motive force was continued until this weaker radiation returned to the original motive wall, the wall was pulled by the radiation and if the external radiation was then removed the whole cavity continued moving at the velocity which it had attained after the completion of this feedback cycle. Jennison and Drinkwater showed that this velocity was approximately twice the velocity at which the motive wall was originally pushed and that, if the application of the motive force was continued, the system would ride up a 'staircase' of velocity. Upon removal of the motive force the system continued to move at the velocity of the last complete quantised state. The units of proper length and proper time defined by this system were conserved and invariant between each quantised velocity state, justifying the assumption built into the framework of both special and general relativity.

The derivation of Newton's laws required only a first-order treatment in view of the magnitude of the primary effect and, although an elementary discussion of the quantisation phenomenon was given, this again was treated at first order in v/c. Furthermore the model of a cavity used in the derivation of Newton's laws was clearly taken as a basis for general illustration of the first-order inertial principle and it was not intended to be a necessary condition that the system should be bounded by macroscopic charged plates. All that is required for a phase-locked cavity is some means of retaining the radiation in a small region of space. The analysis of the standing-wave system then gives the quantised inertial properties of the whole system. Thus it is to be expected that a fundamental particle will comply with these conditions but it will be shown that there may well be only a single node and the binding of the

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energy may result simply from the spinning of the standing-wave system. The model of an electron mentioned in the conclusions of the Jennison and Drinkwater paper was a cavity of this type.

The analysis in this paper includes relativistic effects. It does not presuppose any particular form of binding mechanism for the radiation in the cavity and it will become clear from the analysis that the binding must be accommodated by a suitable configuration of the wave system for no other energy sources are available.

A large number of variations of the basic model are available. Figure 1 shows a set of basic modes plotted rectilinearly and is intended only to simplify discussion of more practical models. A final analysis of a particular particle must obviously include the effects of spin and the three-dimensional distribution of the wave fields which may very well be such as to give rise to re-entry of the internal wave system such that a standing wave to one side of the node loops around and appears also as the balancing standing wave on the other side of the node (e.g. figure 2). In order to assess whether or not such a system is a likely model, the present paper will include, in addition to the simple cavity, the analysis applicable to simple push-pull systems such as the family including open-ended half-wave systems with a central node.

The following symbolism will be used: ν shall refer to frequency; A to amplitude; V and v to velocity (v being used where δv was used by Jennison and Drinkwater). An unprimed amplitude or frequency refers to incident radiation in the laboratory system; a single prime refers to radiation measured in the frame of the node; double and triple primes refer to reflected radiation in the laboratory system; the suffix $_0$ indicates that the quantity is associated with the internal cavity system.

2. Relativistic velocity relationships for a cavity

For a physical account of this process, reference should be made to Jennison and Drinkwater (1977, p 171).

Consider a simple double-noded cavity, such as that in figure 1(a), filled with radiation of frequency ν_0. The cavity is initially at rest in the laboratory.

When the motive reflector element, A, is moved forward with velocity v relative to the laboratory, the internal frequency ν_0 is received at the motive reflecting element at a frequency

$$\nu_0' = \nu_0\left(\frac{1+v/c}{1-v/c}\right)^{1/2}.$$

It is then reflected in the laboratory system at the frequency

$$\nu_0'' = \nu_0'\left(\frac{1+v/c}{1-v/c}\right)^{1/2} = \nu_0\left(\frac{1+v/c}{1-v/c}\right).$$

This radiation moves forward to the reflecting element, B, (the following mirror element) on the right. This element must remain in equilibrium in its own frame still subject to the original restoring force. The following mirror element therefore moves at velocity V_1 such that the frequency is restored to the original value. Hence

$$\nu_0''\left(\frac{1-V_1/c}{1+V_1/c}\right)^{1/2} = \nu_0$$

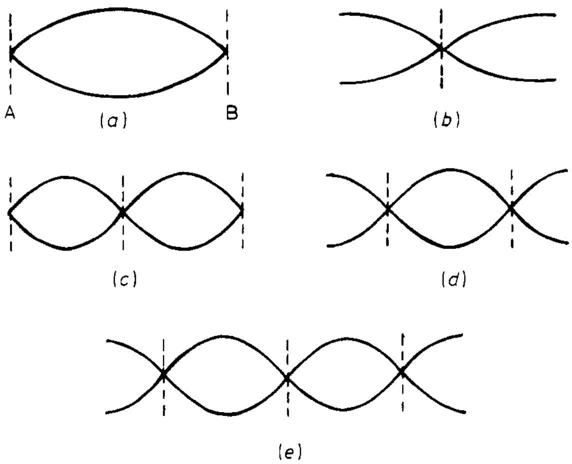

Figure 1. Some basic cavity modes which are discussed in this text. Rectilinear systems are depicted for simplicity, the real systems would be three-dimensional spinning configurations.

therefore

$$\begin{aligned} \nu_0\left(\frac{1+v/c}{1-v/c}\right)\left(\frac{1-V_1/c}{1+V_1/c}\right)^{1/2} &= \nu_0 \\ \frac{1+v/c}{1-v/c} &= \left(\frac{1+V_1/c}{1-V_1/c}\right)^{1/2}. \end{aligned} \tag{1}$$

The frequency reflected back by the following mirror, B, when referred to the laboratory system is

$$\nu_0''' = \nu_0\left(\frac{1-V_1/c}{1+V_1/c}\right)^{1/2} = \nu_0\left(\frac{1-v/c}{1+v/c}\right).$$

This frequency when received in the frame of the moving motive mirror, A, is

$$\nu_0'''\left(\frac{1+v/c}{1-v/c}\right)^{1/2} = \nu_0\left(\frac{1-v/c}{1+v/c}\right)^{1/2}.$$

The effect of this radiation is to cause the motive mirror to move at velocity v relative to its frame of rest at the time. By the relativistic composition of velocities, the new velocity relative to the laboratory frame is then

$$V_2 = \frac{2v}{1+v^2/c^2}. \tag{2}$$

By substitution in equation (1) it will be seen that $V_1 = V_2$ and this quantity will henceforth be referred to as V. Equation (2) is the rigorous expression for the final velocity after the completed acquisition of the first quantum in the process of momentum capture. It was expressed as $2\delta v$ to the first order only in the paper by Jennison and Drinkwater. The velocity relationship between V and v appears to apply to phase-locked cavities of all types. In view of its central importance to the whole analysis it is therefore useful and economical in later calculations to tabulate a number of equivalent forms (equation (3)).

$$\begin{aligned}
&\frac{2v}{1+v^2/c^2}=V\\
&v=\frac{1-(1-V^2/c^2)^{1/2}}{V/c^2}\\
&\frac{1+v/c}{1-v/c}=\left(\frac{1+V/c}{1-V/c}\right)^{1/2}\\
&\frac{1-v^2/c^2}{1+v^2/c^2}=(1-V^2/c^2)^{1/2}.
\end{aligned}\tag{3}$$

In the following discussion it will be assumed that the motive force is provided from an incident wave train of sufficient duration to maintain one reflected excursion of the internal wave. The effect of much longer exposure to the incident radiation has been treated in Jennison and Drinkwater and will be mentioned again in the conclusion of this paper.

The push-pull cavity configuration, in which the central node, when at rest, is balanced by the internal radiation from either side, has much to commend it and some of its properties will now be analysed.

3. Intensity relationships for a push-pull cavity

Many of the relationships for a cavity appear at first to be complicated for they are functions of velocity and, if the cavity is caused to move by the application of an external wave of laboratory frequency ν and amplitude A the velocity is itself a function involving these parameters and the parameters of the cavity. Nevertheless three properties of the system can help in the solution of some of the problems. Firstly the overall action of the cavity is that it relativistically integrates increments of the velocity when the motivation is applied for an extended period. Secondly, it is possible to find certain relationships which are velocity independent although they are derived from parameters which are dependent upon the instantaneous velocity. Finally the various components of force at the node must be such that local equilibrium may be satisfied at all times. If the configuration of the particle gives rise to static electric and magnetic fields in the laboratory then these will couple to an applied electromagnetic field and it will vibrate in sympathy.

Consider a push-pull cavity such as that in figure 1(*b*). When the cavity is at rest, radiation of frequency $\nu_{0\mathrm{L}}$ and amplitude A_0 reaches the central node from the left and radiation of identical frequency $\nu_{0\mathrm{R}}$ and amplitude A_0 reaches the node from the right. The two radiation pressures on an element of the node are identical and the node is in equilibrium. Now apply from the left an external signal of frequency ν and amplitude A in the laboratory. The element starts to move and the signal becomes ν' at the node. At the same time the cavity radiation on the left is doppler-shifted at the node to the value $\nu'_{0\mathrm{L}}$ and that on the right increases to $\nu'_{0\mathrm{R}}$. Thus three signals fall on the same element, two from the left and one from the right. For local equilibrium the zero-frequency components must cancel at the node and therefore

$$(A')^2+(A'_{0L})^2=(A'_{0R})^2$$

which may be written:

$$(A')^2+A_0^2\frac{1-v/c}{1+v/c}=A_0^2\frac{1+v/c}{1-v/c}$$

whence by equation (3)

$$(A')^2=A_0^2\frac{2V/c}{(1-V^2/c^2)^{1/2}} \quad (4)$$

or, in terms of A,

$$A^2=A_0^2\frac{2V/c}{1-V/c}. \quad (5)$$

Now

$$A''=A\left(\frac{1-V/c}{1+V/c}\right)^{1/2}$$

hence

$$A^2-(A'')^2=A^2\left(1-\frac{1-V/c}{1+V/c}\right)$$

and therefore by equation (5)

$$A^2-(A'')^2=A_0^2\left(\frac{4V^2/c^2}{1-V^2/c^2}\right).$$

Otherwise, from (4)

$$A^2-(A'')^2=A_0^2\left(\frac{(A')^2}{A_0^2}\right)^2=\frac{(A')^4}{A_0^2}$$

and hence, at the node, since $(A')^2=AA''$

$$\frac{1}{(A'')^2}-\frac{1}{A^2}=\frac{1}{A_0^2}. \quad (6)$$

4. Spectral relationships

The coupling of an external wave to an element of the node of a cavity produces a steady forward motion fully modulated by an approximately cosinusoidal motion of negative sign and double the signal frequency, in accordance with the classical application of radiation pressure. The motion does not therefore start with a jerk but commences smoothly from zero velocity. The effect of this motion is to frequency modulate the internal radiation in the cavity in a manner very similar to the process of parametric amplification in electronic systems. The motion is opposed by the change in the internal cavity radiation and this opposition has to agree not only for the mean value but also for the rate of change and therefore the periodicity of the signal.

Furthermore the frequency conversion is such that it is perfectly consistent with the required radiation in the cavity in its next quantised state when this radiation is referred to the laboratory system. In the proper frame of the node, on the other hand, the internal frequency reverts to its original proper value in the next quantised state if the external signal is removed.

It was pointed out by Jennison (1977) that the simple double-noded cavity, such as that in figure 1(*a*) presents problems with the symmetrical application of the reciprocity theorem to both nodes. The present analysis shows that problems also arise if one endeavours to associate the spectral relationships of a double-noded cavity with the experimentally observed properties of an electron. These problems do not arise however if the electron is modelled on the principle of a single-noded push-pull cavity, such as in figure 1(*b*). A very simple treatment is available from elementary systems analysis. The spectrum of frequencies produced in the cavity when it is pushed can be simplified considerably if $\nu'_{0R} - \nu'_{0L} = 2\nu'$. Looking at this from the point of view of the mechanics it can be seen that to satisfy local equilibrium at an element of the reflective node, the rates of fluctuation of the radiation pressures from the separate wave systems inside and outside the cavity must agree. Then the difference frequency $\nu'_{0R} - \nu'_{0L}$ must equal the second harmonic of ν'. Therefore

$$\nu'_{0R} - \nu'_{0L} = 2\nu'. \tag{7}$$

The cavity frequencies ν'_{0R} and ν'_{0L} are related to the cavity rest frequency by the same instantaneous velocity v and so

$$\nu'_{0R} - \nu'_{0L} = \nu_0\left(\frac{1+v/c}{(1-v^2/c^2)^{1/2}} - \frac{1-v/c}{(1-v^2/c^2)^{1/2}}\right) = \nu_0\frac{2v/c}{(1-v^2/c^2)^{1/2}}$$

hence

$$\nu' = \nu_0\frac{v/c}{(1-v^2/c^2)^{1/2}}. \tag{8}$$

This is the relativistic form of the equation

$$\nu_a = \nu_0\frac{\overline{\delta v}}{c}$$

of Jennison and Drinkwater (1977, p 173), in which ν_a was used as the symbol for ν' and $\overline{\delta v}$ was used for the velocity v in this account.

Equations (3) and (8) are of considerable significance for the elucidation of quantum phenomena and the modelling of fundamental particles. This will now be illustrated by considering the behaviour of the electron in the Compton effect.

5. Derivation of the Compton back-scatter energy equation for a push-pull cavity

If ν is the incident external radiation in the laboratory giving rise to ν' at the node and if ν'' is the corresponding reflected radiation in the laboratory,

$$\nu - \nu'' = \nu'\frac{2v/c}{(1-v^2/c^2)^{1/2}}.$$

Therefore from equation (8)

$$\frac{\nu-\nu''}{\nu_0}=\frac{2v^2/c^2}{1-v^2/c^2}$$

and therefore from equation (3)

$$\frac{\nu-\nu''}{\nu_0}=\frac{v}{c}\frac{V/c}{(1-V^2/c^2)^{1/2}}=\frac{1}{(1-V^2/c^2)^{1/2}}-1.$$

Multiplying both sides by m_0c^2, the rest energy of the cavity derived in the paper by Jennison and Drinkwater (1977),

$$\frac{m_0c^2}{\nu_0}(\nu-\nu'')=\frac{m_0c^2}{(1-V^2/c^2)^{1/2}}-m_0c^2. \tag{9}$$

But m_0c^2/ν_0 is a proper constant of a cavity formed from the annihilation frequency ν_0 of the electron and has the same value as Planck's constant, h. (The expression may be written $2m_0c^2/\nu_0$ if the frequency is specified as the pair production frequency as in Jennison and Drinkwater). Hence

$$h\nu-h\nu''=\frac{m_0c^2}{(1-V^2/c^2)^{1/2}}-m_0c^2 \tag{10}$$

which is the Compton back-scatter energy relationship. The expression on the right is the kinetic energy of a mass m_0 recoiling at velocity V and that on the left is the photon energy which is inelastically absorbed in the impact.

6. Derivation of the momentum relationship for Compton back-scatter of an electron

It is of interest to show that the equations also give the correct result for the quantised momentum.

$$\nu+\nu''=\nu'\left(\frac{1-v/c}{1+v/c}\right)^{1/2}+\nu'\left(\frac{1+v/c}{1-v/c}\right)^{1/2}=2\nu'\frac{1}{(1-v^2/c^2)^{1/2}}.$$

Thus from equation (8)

$$\nu+\nu''=\nu_0\frac{2v/c}{(1-v^2/c^2)^{1/2}}\frac{1}{(1-v^2/c^2)^{1/2}}=\nu_0\frac{2v/c}{1-v^2/c^2} \tag{11}$$

and therefore from equation (3)

$$\nu+\nu''=\nu_0\frac{V/c}{(1-V^2/c^2)^{1/2}} \tag{12}$$

$$\frac{m_0c}{\nu_0}(\nu+\nu'')=\frac{m_0V}{(1-V^2/c^2)^{1/2}} \tag{13}$$

and therefore

$$\frac{h\nu}{c}+\frac{h\nu''}{c}=\frac{m_0V}{(1-V^2/c^2)^{1/2}}$$

which is the correct quantum relationship for the momentum.

7. Conclusions

This paper has dealt with the previously unexplored problem of the process during the acquisition of a new quantum state by a free electron. It shows that this can come about entirely from the transfer function of the particle even when the applied radiation is an arbitrary classical electromagnetic wave, provided that it is applied for a time which exceeds the capture time of the cavity. If the radiation lasts somewhat longer, the excess will be re-radiated by reciprocity as explained by Jennison and Drinkwater. If the applied radiation lasts for approximately twice the time it may raise the particle to the next quantised velocity state but the second increment will be smaller, as the recoiling particle will receive radiation which is Doppler shifted by its own motion. Though this account has treated the case of a *free* electron there is no reason why similar principles should not be applied to electrons orbiting in a central field of electric force.

A classical treatment of the angular relationships in the Compton effect has been given by Ashworth and Jennison (1974) in terms of the velocity V. The present paper shows how V is related to the velocity v and how this leads to a classical derivation of the quantised energy and momentum for the particular case of back-scatter. A full classical solution of the Compton effect should now be possible in terms of the velocity v. Ashworth has analysed Compton angular scattering in the context of reflection from a mirror moving at the mean velocity v and his results are entirely compatible with the concept of a phase-locked cavity.

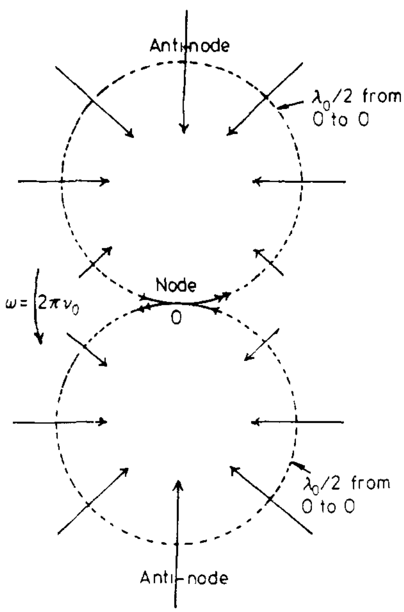

Figure 2. One possible model of a single-noded re-entrant push-pull cavity. The phase of the wave reverses in synchronism with the rotation and thus the electric field vectors always point inwards (outwards for a cavity with reversed spin). The electromagnetic wave loops on itself, the electric field has a static radial component and the magnetic field has a dipole component through the centre. Combination with a similar system in quadrature (not shown) could produce a purely static, although spinning, field system in the laboratory. The system is equivalent to a simple standing wave locked into a rotating frame.

The analysis in this paper provides some insight into the structure of the electron. The choice of basic cavity is reduced to that in figure 1(*b*) in which the electric field vanishes at the centre where the magnetic field is the strongest. Figure 1(*b*) is nevertheless only diagrammatic whereas the radiation must be trapped, three-dimensional, and spinning. A possible configuration satisfying these requirements is shown in figure 2. This configuration may have an associated orthogonal wave system and would appear in the laboratory as a localised monopolar divergence of electric field without a central infinity and with a magnetic dipole through the centre. The whole system spins and the field carries angular momentum which is a proper constant referred to the centre of the system in any inertial frame.

The remaining basic configurations of figure 1 are of some interest. Figures 1(*a*) and 1(*d*) have a maximum electric field at the centre and a spinning configuration with an axis through the centre presents severe difficulties with the central singularity. Figure 1(*c*) could be spun about the central node and would appear to have no external electric field but it is not yet clear if the reciprocity property can be satisfied at the outer nodes unless they represent points on a continuous nodal surface spinning at the velocity of light. Figure 1(*e*) can be spun about the centre and it again has an external electric field. It is somewhat tempting to identify the outer nodes with a 'hard' surface unlike the 'softer' nature of the outer electric field and it is also interesting that the whole system in figure 1(*e*) can be looked upon as having three constituent but inseparable parts. It is tempting to consider these parts as quarks, the whole corresponding to the proton. A self-contained, three-dimensional, spinning model of figure 1(*e*) has not yet been propounded but the frequency for this mode to be trapped by a natural process will presumably differ from that applicable in figure 1(*b*).

Acknowledgment

The author is indebted to Mr A J Drinkwater for an earlier derivation of equation (2).

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